**Name: Vijaya Sanjayrao Pawar**

**Q. Given data set on revenue and sales quantity of a beverage manufacturer between Jan 2015 to Apr 2020.Suggest appropriate smoothing technique for the series. Compare between the revenue and the average cost. Does the same technique work for both, Justify.**

The problem suggests analyzing and comparing revenue and average cost by performing any smoothing technique .We need to measure the underlying trend and as observed revenue showcases trend with seasonality whereas average cost shows only seasonality.

Exponential smoothing is a [rule of thumb](https://en.wikipedia.org/wiki/Rule_of_thumb) technique for smoothing [time series](https://en.wikipedia.org/wiki/Time_series) data using the exponential [window function](https://en.wikipedia.org/wiki/Window_function). Whereas in the [simple moving average](https://en.wikipedia.org/wiki/Simple_moving_average) the past observations are weighted equally, exponential functions are used to assign exponentially decreasing weights over time. It is an easily learned and easily applied procedure for making some determination based on prior assumptions by the user, such as seasonality. Exponential smoothing is often used for analysis of time-series data.

Exponential smoothing was first suggested in the statistical literature without reference to previous work by Robert Goodell Brown in 1956 and then expanded by Charles C. Holt in 1957. Exponential smoothing is a broadly accurate principle for smoothing time series data using the exponential window function. The controlling input of the exponential smoothing calculation is defined as the smoothing factor or the smoothing constant.

As we know that, in the simple moving average, the past observations are weighted equally, [exponential functions](https://byjus.com/maths/exponential-functions/) are used to assign exponentially decreasing weights over time. It is an easily learned and easily applied method for making some determination based on prior assumptions by the user, such as seasonality. Exponential smoothing is generally used for the analysis of time-series data.

Exponential Smoothing Formula

The simplest form of an exponential smoothing formula is given by:

st = αxt+(1 – α)st-1= st-1+ α(xt– st-1)

Here,

st = smoothed statistic, it is the simple weighted average of current observation xt

st-1 = previous smoothed statistic

α = smoothing factor of data; 0 < α < 1

t = time period

If the value of the smoothing factor is larger, then the level of smoothing will reduce. Value of α close to 1 has less of a smoothing effect and give greater weight to recent changes in the data, while the value of α closer to zero has a greater smoothing effect and are less responsive to recent changes.

There is no official accurate procedure for choosing α. The statistician’s judgment is used to choose an appropriate factor sometimes. Otherwise, a statistical technique may be used to optimize the value of α. For example, the method of least squares can be used to determine the value of α for which the sum of the quantities is minimized.

Exponential Smoothing Forecasting

Exponential smoothing is generally used to make short term forecasts, but longer-term forecasts using this technique can be quite unreliable. More recent observations given larger weights by exponential smoothing methods, and the weights decrease exponentially as the observations become more distant. When the parameters describing the time series are changing slowly over time, then these methods are most effective.

Exponential Smoothing Methods

There are three main methods to estimate exponential smoothing. They are:

* Simple or single exponential smoothing
* Double exponential smoothing
* Triple exponential smoothing

Simple or single exponential smoothing

If the data has no trend and no seasonal pattern, then this method of forecasting the time series is essentially used. This method uses weighted moving averages with exponentially decreasing weights.

The single exponential smoothing formula is given by:

st= αxt+(1 – α)st-1 = st-1 + α(xt – st-1)

Double exponential smoothing

This method is also called as Holt’s trend corrected or second-order exponential smoothing. This method is used for forecasting the time series when the data has a linear trend and no seasonal pattern. The primary idea behind double exponential smoothing is to introduce a term to take into account the possibility of a series showing some form of trend. This slope component is itself updated through exponential smoothing.

The double exponential smoothing formulas are given by:

S1 = x1

B1 = x1-x0

For t>1,

st = αxt + (1 – α)(st-1 + bt-1)

βt = β(st – st-1) + (1 – β)bt-1

Here,

st = smoothed statistic, it is the simple weighted average of current observation xt

st-1 = previous smoothed statistic

α = smoothing factor of data; 0 < α < 1

t = time period

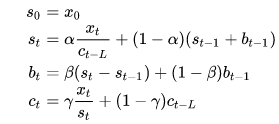
bt = best estimate of trend at time t

β = trend smoothing factor; 0 < β <1

Triple exponential smoothing

In this method, exponential smoothing applied three times. This method is used for forecasting the time series when the data has both linear trend and seasonal pattern. This method is also called Holt-Winters exponential smoothing.

The triple exponential smoothing formulas are given by:



Here,

st = smoothed statistic, it is the simple weighted average of current observation xt

st-1 = previous smoothed statistic

α = smoothing factor of data; 0 < α < 1

t = time period

bt = best estimate of a trend at time t

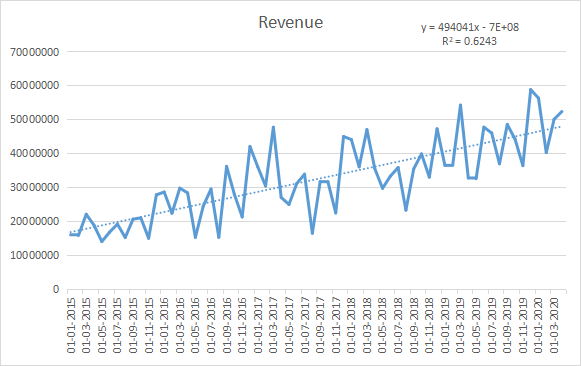
β = trend smoothing factor; 0 < β <1

ct = sequence of seasonal correction factor at time t

γ = seasonal change smoothing factor; 0 < γ < 1

**Analyse:-**

1)To see whether there is a trend that is been followed since Jan 2015 to Apr 2020 , a line chart is plotted as we can clearly see the trend with seasonality in it over a period of 12 months as there is both seasonality and trend in this particular quantity we can use Holt’s winter smoothing as double smoothing will not work in case of data contain seasonality. So that for smoothing the seasonality a third equation is introduced.



By taking α Data smoothing factor. The range is 0 < α <1.

β Trend smoothing factor. The range is 0 < β < 1.

γ Seasonal change smoothing factor. The range is 0 < γ <1

Here these values turned out to be as follows after using the solver function in excel whereas the MAPE is

|  |  |  |
| --- | --- | --- |
| alpha | beta | gamma |
| 0.80441 | 0 | 0.19559 |

And the smoothened values are

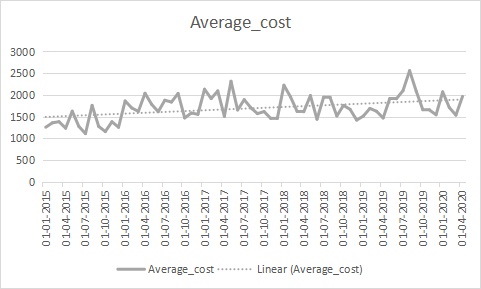
|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Period | Revenue | Lt | Bt | St | Yt | ERROR |
| 01-01-2015 | 16010072 |  |  | 0.072042 |  |  |
| 01-02-2015 | 15807587 |  |  | 0.07113 |  |  |
| 01-03-2015 | 22047146 |  |  | 0.099207 |  |  |
| 01-04-2015 | 18814583 |  |  | 0.084661 |  |  |
| 01-05-2015 | 14021480 |  |  | 0.063093 |  |  |
| 01-06-2015 | 16783929 |  |  | 0.075524 |  |  |
| 01-07-2015 | 19161892 |  |  | 0.086224 |  |  |
| 01-08-2015 | 15204984 |  |  | 0.068419 |  |  |
| 01-09-2015 | 20603940 |  |  | 0.092713 |  |  |
| 01-10-2015 | 20992875 |  |  | 0.094463 |  |  |
| 01-11-2015 | 14993370 |  |  | 0.067467 |  |  |
| 01-12-2015 | 27791808 | 22356000 | 0 | 0.125057 |  |  |
| 01-01-2016 | 28601586 | 27380010 | 0 | 1221576 | 28601586 | 8.98E-08 |
| 01-02-2016 | 22367074 | 23347556 | 0 | -980482 | 22367074 | 9.22E-08 |
| 01-03-2016 | 29738609 | 28488581 | 0 | 1250028 | 29738609 | 8.84E-08 |
| 01-04-2016 | 28351008 | 28377916 | 0 | -26907.8 | 28351008 | 2E-09 |
| 01-05-2016 | 15264604 | 17829440 | 0 | -2564836 | 15264604 | 3.53E-07 |
| 01-06-2016 | 24385658 | 23103325 | 0 | 1282333 | 24385658 | 1.11E-07 |
| 01-07-2016 | 29486517 | 28238027 | 0 | 1248490 | 29486517 | 8.91E-08 |
| 01-08-2016 | 15270117 | 17806514 | 0 | -2536397 | 15270117 | 3.49E-07 |
| 01-09-2016 | 36141028 | 32554975 | 0 | 3586053 | 36141027 | 2.09E-07 |
| 01-10-2016 | 27915144 | 28822649 | 0 | -907506 | 27915144 | 6.84E-08 |
| 01-11-2016 | 21272049 | 22748873 | 0 | -1476824 | 21272049 | 1.46E-07 |
| 01-12-2016 | 42014160 | 38246057 | 0 | 3768103 | 42014160 | 1.89E-07 |
| 01-01-2017 | 36007381 | 35462596 | 0 | 544784.4 | 36007381 | 3.95E-08 |
| 01-02-2017 | 30396775 | 32176310 | 0 | -1779534 | 30396775 | 5.53E-08 |
| 01-03-2017 | 47678131 | 43640591 | 0 | 4037540 | 47678131 | 1.23E-07 |
| 01-04-2017 | 27013965 | 30287616 | 0 | -3273651 | 27013965 | 2.53E-07 |
| 01-05-2017 | 24948845 | 28056236 | 0 | -3107391 | 24948845 | 4.57E-08 |
| 01-06-2017 | 31101346 | 29474231 | 0 | 1627115 | 31101346 | 2.33E-08 |
| 01-07-2017 | 33848822 | 31988897 | 0 | 1859925 | 33848822 | 3.8E-08 |
| 01-08-2017 | 16454667 | 21533314 | 0 | -5078647 | 16454667 | 3.25E-07 |
| 01-09-2017 | 31650093 | 26786693 | 0 | 4863399 | 31650093 | 8.49E-08 |
| 01-10-2017 | 31572206 | 31366212 | 0 | 205993.2 | 31572206 | 7.42E-08 |
| 01-11-2017 | 22446371 | 25378977 | 0 | -2932606 | 22446371 | 1.36E-07 |
| 01-12-2017 | 44966126 | 38103971 | 0 | 6862155 | 44966126 | 1.45E-07 |
| 01-01-2018 | 44067521 | 42462879 | 0 | 1604642 | 44067521 | 5.06E-08 |
| 01-02-2018 | 36020287 | 38711870 | 0 | -2691583 | 36020287 | 5.33E-08 |
| 01-03-2018 | 46995990 | 42127860 | 0 | 4868130 | 46995990 | 3.72E-08 |
| 01-04-2018 | 35536488 | 39459053 | 0 | -3922565 | 35536488 | 3.84E-08 |
| 01-05-2018 | 29699599 | 34108069 | 0 | -4408470 | 29699599 | 9.21E-08 |
| 01-06-2018 | 33261065 | 32117864 | 0 | 1143201 | 33261065 | 3.06E-08 |
| 01-07-2018 | 35826535 | 33605013 | 0 | 2221522 | 35826535 | 2.12E-08 |
| 01-08-2018 | 23268655 | 29375659 | 0 | -6107004 | 23268655 | 9.3E-08 |
| 01-09-2018 | 35423490 | 30328427 | 0 | 5095063 | 35423490 | 1.38E-08 |
| 01-10-2018 | 39831566 | 37807141 | 0 | 2024424 | 39831566 | 9.6E-08 |
| 01-11-2018 | 32999145 | 36298559 | 0 | -3299414 | 32999145 | 2.34E-08 |
| 01-12-2018 | 47221828 | 39565359 | 0 | 7656469 | 47221828 | 3.54E-08 |
| 01-01-2019 | 36459960 | 35776556 | 0 | 683403.9 | 36459960 | 5.31E-08 |
| 01-02-2019 | 36546499 | 38561041 | 0 | -2014542 | 36546499 | 3.9E-08 |
| 01-03-2019 | 54198707 | 47224160 | 0 | 6974547 | 54198707 | 8.17E-08 |
| 01-04-2019 | 32743990 | 38731520 | 0 | -5987530 | 32743990 | 1.33E-07 |
| 01-05-2019 | 32531658 | 37290506 | 0 | -4758849 | 32531658 | 2.27E-08 |
| 01-06-2019 | 47709702 | 44752206 | 0 | 2957495 | 47709702 | 8E-08 |
| 01-07-2019 | 45992142 | 43962608 | 0 | 2029533 | 45992142 | 8.78E-09 |
| 01-08-2019 | 36933665 | 43220991 | 0 | -6287326 | 36933665 | 1.03E-08 |
| 01-09-2019 | 48526260 | 43390083 | 0 | 5136177 | 48526260 | 1.78E-09 |
| 01-10-2019 | 44160416 | 42381280 | 0 | 1779136 | 44160416 | 1.17E-08 |
| 01-11-2019 | 36374956 | 40203816 | 0 | -3828859 | 36374957 | 3.06E-08 |
| 01-12-2019 | 58756474 | 48968816 | 0 | 9787658 | 58756474 | 7.63E-08 |
| 01-01-2020 | 56288301 | 54306944 | 0 | 1981357 | 56288301 | 4.85E-08 |
| 01-02-2020 | 40225243 | 44600004 | 0 | -4374761 | 40225243 | 1.23E-07 |
| 01-03-2020 | 50022165 | 43351250 | 0 | 6670915 | 50022165 | 1.28E-08 |
| 01-04-2020 | 52320693 | 55382785 | 0 | -3062092 | 52320693 | 1.18E-07 |
|  |  |  |  |  | MAPE | 8.78E-08 |

Graph of smoothed values and actual values shows

Holt’s winter smoothing is appropriate for revenue

The higher a smoothing constant, the more sensitive your demand forecast. This means we will see large spikes of data. This is what a smoothing constant of 0.80441 would look like with our data.

2)



The next quantity average cost of the beverage is analysed and as the line chart show clearly there is a trend but the slope of the line is relatively small.

So here we use double smoothing technique.

|  |  |
| --- | --- |
| alpha | 0.8 |
| beta | 0.5659884 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Period | Average\_cost | st | Bt | Yt | error |
| 01-01-2015 | 1257.763541 | 1257.764 | 100.7435 |  |  |
| 01-02-2015 | 1358.507 | 1318.21 | 77.93561 | 1318.21 | 2.966299 |
| 01-03-2015 | 1384.697024 | 1363.872 | 59.66928 | 1363.872 | 1.503948 |
| 01-04-2015 | 1235.606705 | 1253.491 | -36.5772 | 1253.491 | 1.447403 |
| 01-05-2015 | 1626.621765 | 1555.734 | 155.1913 | 1555.734 | 4.357963 |
| 01-06-2015 | 1275.374508 | 1314.586 | -69.1324 | 1314.586 | 3.074485 |
| 01-07-2015 | 1110.576805 | 1157.363 | -118.991 | 1157.363 | 4.212768 |
| 01-08-2015 | 1759.42887 | 1653.457 | 229.14 | 1653.457 | 6.023108 |
| 01-09-2015 | 1276.259909 | 1327.066 | -85.284 | 1327.066 | 3.980834 |
| 01-10-2015 | 1157.588904 | 1198.38 | -109.849 | 1198.38 | 3.52379 |
| 01-11-2015 | 1383.024597 | 1359.907 | 43.74689 | 1359.907 | 1.671507 |
| 01-12-2015 | 1256.808558 | 1273.302 | -30.0307 | 1273.302 | 1.312358 |
| 01-01-2016 | 1861.476505 | 1746.549 | 254.8184 | 1746.549 | 6.173994 |
| 01-02-2016 | 1700.530226 | 1681.756 | 73.92193 | 1681.756 | 1.104034 |
| 01-03-2016 | 1621.604699 | 1622.605 | -1.39545 | 1622.605 | 0.061712 |
| 01-04-2016 | 2038.321083 | 1955.257 | 187.6712 | 1955.257 | 4.075128 |
| 01-05-2016 | 1784.707557 | 1797.896 | -7.61293 | 1797.896 | 0.73897 |
| 01-06-2016 | 1614.837301 | 1650.334 | -86.8225 | 1650.334 | 2.198156 |
| 01-07-2016 | 1878.720425 | 1843.308 | 71.53926 | 1843.308 | 1.884906 |
| 01-08-2016 | 1836.675157 | 1830.776 | 23.95594 | 1830.776 | 0.321167 |
| 01-09-2016 | 2034.509545 | 1990.151 | 100.6016 | 1990.151 | 2.180283 |
| 01-10-2016 | 1471.61915 | 1564.077 | -197.491 | 1564.077 | 6.282723 |
| 01-11-2016 | 1583.566541 | 1600.675 | -64.9991 | 1600.675 | 1.080392 |
| 01-12-2016 | 1554.410444 | 1573.241 | -43.7376 | 1573.241 | 1.211459 |
| 01-01-2017 | 2132.001934 | 2025.231 | 236.8383 | 2025.231 | 5.008006 |
| 01-02-2017 | 1916.085185 | 1911.901 | 38.64695 | 1911.901 | 0.218378 |
| 01-03-2017 | 2092.43091 | 2049.432 | 94.61445 | 2049.432 | 2.054956 |
| 01-04-2017 | 1508.317405 | 1606.217 | -209.791 | 1606.217 | 6.490664 |
| 01-05-2017 | 2315.008323 | 2195.628 | 242.5482 | 2195.628 | 5.156785 |
| 01-06-2017 | 1654.414891 | 1738.024 | -153.73 | 1738.024 | 5.053693 |
| 01-07-2017 | 1891.101303 | 1874.51 | 10.52892 | 1874.51 | 0.877334 |
| 01-08-2017 | 1705.323552 | 1740.373 | -71.3501 | 1740.373 | 2.055315 |
| 01-09-2017 | 1570.02295 | 1611.353 | -103.991 | 1611.353 | 2.632455 |
| 01-10-2017 | 1617.511431 | 1628.812 | -35.2517 | 1628.812 | 0.698632 |
| 01-11-2017 | 1461.35228 | 1499.634 | -88.4126 | 1499.634 | 2.61964 |
| 01-12-2017 | 1458.376602 | 1476.654 | -51.3786 | 1476.654 | 1.253287 |
| 01-01-2018 | 2224.284315 | 2081.378 | 319.968 | 2081.378 | 6.4248 |
| 01-02-2018 | 1955.074205 | 1944.923 | 61.63739 | 1944.923 | 0.519242 |
| 01-03-2018 | 1620.3279 | 1674.95 | -126.05 | 1674.95 | 3.371033 |
| 01-04-2018 | 1612.875581 | 1639.576 | -74.7283 | 1639.576 | 1.65546 |
| 01-05-2018 | 1985.400039 | 1925.841 | 129.5896 | 1925.841 | 2.99986 |
| 01-06-2018 | 1441.932865 | 1524.708 | -170.793 | 1524.708 | 5.740595 |
| 01-07-2018 | 1947.411801 | 1880.475 | 127.2334 | 1880.475 | 3.437239 |
| 01-08-2018 | 1931.810312 | 1909.484 | 71.63973 | 1909.484 | 1.155724 |
| 01-09-2018 | 1516.546359 | 1585.271 | -152.408 | 1585.271 | 4.531668 |
| 01-10-2018 | 1759.033991 | 1741.018 | 22.00402 | 1741.018 | 1.024193 |
| 01-11-2018 | 1669.574764 | 1683.066 | -23.2503 | 1683.066 | 0.808053 |
| 01-12-2018 | 1422.044394 | 1476.201 | -127.174 | 1476.201 | 3.80833 |
| 01-01-2019 | 1513.112554 | 1520.334 | -30.2162 | 1520.334 | 0.477243 |
| 01-02-2019 | 1690.089653 | 1660.737 | 66.35268 | 1660.737 | 1.736723 |
| 01-03-2019 | 1623.736682 | 1623.737 | 7.855849 | 1623.737 | 3.57E-06 |
| 01-04-2019 | 1470.648534 | 1499.695 | -66.7967 | 1499.695 | 1.975078 |
| 01-05-2019 | 1917.348827 | 1841.368 | 164.3925 | 1841.368 | 3.962801 |
| 01-06-2019 | 1911.599553 | 1879.871 | 93.14039 | 1879.871 | 1.659795 |
| 01-07-2019 | 2098.46884 | 2042.467 | 132.4515 | 2042.467 | 2.668705 |
| 01-08-2019 | 2559.328184 | 2440.666 | 282.8615 | 2440.666 | 4.636457 |
| 01-09-2019 | 2086.881698 | 2124.799 | -56.012 | 2124.799 | 1.816921 |
| 01-10-2019 | 1659.978806 | 1756.562 | -232.728 | 1756.562 | 5.818327 |
| 01-11-2019 | 1654.384704 | 1702.049 | -131.86 | 1702.049 | 2.881093 |
| 01-12-2019 | 1543.420464 | 1591.985 | -119.524 | 1591.985 | 3.146574 |
| 01-01-2020 | 2070.64085 | 1989.101 | 172.8885 | 1989.101 | 3.93788 |
| 01-02-2020 | 1711.057181 | 1748.396 | -61.2008 | 1748.396 | 2.18222 |
| 01-03-2020 | 1535.882748 | 1583.158 | -120.085 | 1583.158 | 3.078037 |
| 01-04-2020 | 1965.83479 | 1903.861 | 129.3963 | 1903.861 | 3.152525 |
|  |  |  |  | MAPE | 2.7637 |

3)The next given quantity is sales quantity as the graph depicts a very clear increasing trend with some seasonality Triple exponential smoothing will be preferred to smooth and see the underlying trend,

|  |  |
| --- | --- |
| alpha | 0.723049547 |
| beta | 9.85467E-07 |
| gamma | 0.276951971 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Period | Sales\_quantity | Lt | bt | St | yt | error |
| 01-01-2015 | 12729 |  |  | 0.074691 |  |  |
| 01-02-2015 | 11636 |  |  | 0.067237 |  |  |
| 01-03-2015 | 15922 |  |  | 0.091204 |  |  |
| 01-04-2015 | 15227 |  |  | 0.086032 |  |  |
| 01-05-2015 | 8620 |  |  | 0.049068 |  |  |
| 01-06-2015 | 13160 |  |  | 0.07494 |  |  |
| 01-07-2015 | 17254 |  |  | 0.097179 |  |  |
| 01-08-2015 | 8642 |  |  | 0.049105 |  |  |
| 01-09-2015 | 16144 |  |  | 0.091904 |  |  |
| 01-10-2015 | 18135 |  |  | 0.102295 |  |  |
| 01-11-2015 | 10841 |  |  | 0.060865 |  |  |
| 01-12-2015 | 22113 | 15988.74 | 0.015756 | 0.122369 |  |  |
| 01-01-2016 | 15365 | 15537.7 | 0.015312 | -172.697 | 15365.01 | 9.35E-05 |
| 01-02-2016 | 13153 | 13813.4 | 0.013613 | -660.402 | 13153.01 | 7.6E-05 |
| 01-03-2016 | 18339 | 17085.57 | 0.016837 | 1253.437 | 18339.02 | 0.000129 |
| 01-04-2016 | 13909 | 14788.7 | 0.014574 | -879.7 | 13909.01 | 7.01E-05 |
| 01-05-2016 | 8553 | 10279.95 | 0.01013 | -1726.96 | 8553.001 | 7.77E-06 |
| 01-06-2016 | 15101 | 13765.76 | 0.013566 | 1335.251 | 15101.02 | 0.000138 |
| 01-07-2016 | 15695 | 15160.63 | 0.01494 | 534.3745 | 15695.02 | 0.000114 |
| 01-08-2016 | 8314 | 10210.15 | 0.010062 | -1896.16 | 8314 | 3.99E-06 |
| 01-09-2016 | 17764 | 15671.89 | 0.015444 | 2092.119 | 17764.03 | 0.000151 |
| 01-10-2016 | 18969 | 18055.8 | 0.017793 | 913.21 | 18969.02 | 0.00012 |
| 01-11-2016 | 13433 | 14713.25 | 0.014499 | -1280.25 | 13433.01 | 5.57E-05 |
| 01-12-2016 | 27029 | 23618.06 | 0.023275 | 3410.957 | 27029.04 | 0.000155 |
| 01-01-2017 | 16889 | 18877.49 | 0.018603 | -1988.5 | 16889.01 | 5.12E-05 |
| 01-02-2017 | 15864 | 17176.1 | 0.016926 | -1312.1 | 15864.01 | 8.42E-05 |
| 01-03-2017 | 22786 | 20326.04 | 0.02003 | 2459.964 | 22786.03 | 0.000117 |
| 01-04-2017 | 17910 | 19215.2 | 0.018936 | -1305.2 | 17910.02 | 9.27E-05 |
| 01-05-2017 | 10777 | 14362.64 | 0.014154 | -3585.65 | 10777 | 3.68E-05 |
| 01-06-2017 | 18799 | 16604.9 | 0.016363 | 2194.105 | 18799.02 | 0.000112 |
| 01-07-2017 | 17899 | 17154.22 | 0.016905 | 744.7775 | 17899.02 | 0.000101 |
| 01-08-2017 | 9649 | 13098.59 | 0.012908 | -3449.6 | 9649.004 | 4.55E-05 |
| 01-09-2017 | 20159 | 16690.92 | 0.016448 | 3468.092 | 20159.02 | 0.000119 |
| 01-10-2017 | 19519 | 18075.47 | 0.017812 | 1443.534 | 19519.02 | 0.000106 |
| 01-11-2017 | 15360 | 17037.74 | 0.01679 | -1677.74 | 15360.01 | 9.51E-05 |
| 01-12-2017 | 30833 | 24546.11 | 0.024189 | 6286.905 | 30833.04 | 0.00013 |
| 01-01-2018 | 19812 | 22560.91 | 0.022233 | -2748.91 | 19812.02 | 9.12E-05 |
| 01-02-2018 | 18424 | 20518.44 | 0.02022 | -2094.44 | 18424.02 | 8.65E-05 |
| 01-03-2018 | 29004 | 24875.25 | 0.024513 | 4128.76 | 29004.03 | 0.000116 |
| 01-04-2018 | 22033 | 23763.89 | 0.023418 | -1730.89 | 22033.02 | 9.57E-05 |
| 01-05-2018 | 14959 | 19990.13 | 0.019699 | -5031.14 | 14959.01 | 7.87E-05 |
| 01-06-2018 | 23067 | 20628.42 | 0.020328 | 2438.583 | 23067.02 | 9.39E-05 |
| 01-07-2018 | 18397 | 18476.49 | 0.018207 | -79.4914 | 18397.01 | 7.44E-05 |
| 01-08-2018 | 12045 | 16320.44 | 0.016083 | -4275.45 | 12045.01 | 9.59E-05 |
| 01-09-2018 | 23358 | 18901.35 | 0.018626 | 4456.658 | 23358.02 | 0.000103 |
| 01-10-2018 | 22644 | 20563.73 | 0.020264 | 2080.274 | 22644.02 | 0.000105 |
| 01-11-2018 | 19765 | 21199.31 | 0.020891 | -1434.3 | 19765.02 | 0.000112 |
| 01-12-2018 | 33207 | 25335.73 | 0.024967 | 7871.283 | 33207.03 | 0.000101 |
| 01-01-2019 | 24096 | 26426.95 | 0.026042 | -2330.95 | 24096.03 | 0.000118 |
| 01-02-2019 | 21624 | 24468.57 | 0.024112 | -2844.57 | 21624.02 | 9.25E-05 |
| 01-03-2019 | 33379 | 27925.96 | 0.027519 | 5453.046 | 33379.03 | 0.000104 |
| 01-04-2019 | 22265 | 25084.34 | 0.024719 | -2819.34 | 22265.02 | 8.42E-05 |
| 01-05-2019 | 16967 | 22852.87 | 0.02252 | -5885.87 | 16967.02 | 0.000105 |
| 01-06-2019 | 24958 | 22611.77 | 0.022282 | 2346.227 | 24958.02 | 8.73E-05 |
| 01-07-2019 | 21917 | 22166.9 | 0.021844 | -249.901 | 21917.02 | 9.54E-05 |
| 01-08-2019 | 14431 | 19664.83 | 0.019378 | -5233.83 | 14431.01 | 9.79E-05 |
| 01-09-2019 | 23253 | 19036.87 | 0.018759 | 4216.124 | 23253.02 | 7.5E-05 |
| 01-10-2019 | 26603 | 23003.42 | 0.022668 | 3599.586 | 26603.03 | 0.000117 |
| 01-11-2019 | 21987 | 23305.58 | 0.022966 | -1318.58 | 21987.02 | 0.000107 |
| 01-12-2019 | 38069 | 28288.94 | 0.027877 | 9780.068 | 38069.04 | 0.000101 |
| 01-01-2020 | 27184 | 29175.41 | 0.02875 | -1991.41 | 27184.03 | 0.000113 |
| 01-02-2020 | 23509 | 27135.09 | 0.02674 | -3626.1 | 23509.02 | 9.55E-05 |
| 01-03-2020 | 32569 | 27121.26 | 0.026726 | 5447.738 | 32569.03 | 8.2E-05 |
| 01-04-2020 | 26615 | 28793.74 | 0.028374 | -2178.74 | 26615.03 | 0.00012 |
|  |  |  |  |  | MAPE | 9.53E-05 |

Interpretation:-

Based on the all graphs we can say that all three quantities I.e sales quantity , average cost and renvenue of the given data set undergoes some trend .

As all three quantities shows trend in there values revenue and sales quantity shows a increasing trend with seasonality , as the average cost shows trend with a very small slope it is appropriate to use double exponential smoothing to get better smooth values .

Conclusion:-

Recommendation is to use some basic test of autocorrelation to see whether there is evidence of non-zero autocorrelation. Triple exponential smoothing as well as some other models in time series can be used to get more smooth values .